MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2015/2016

PPS0016 – INTRODUCTION TO PROBABILITY AND STATISTICS

(All groups)

13 OCTOBER 2015
2.30 pm – 4.30 pm
(2 Hours)

INSTRUCTION TO STUDENT

1. This examination paper consists of **5 pages** (including the cover page) with **5 questions only**.
2. Each question is worth **20 marks. Attempt ALL questions.**
3. A normal distribution table is provided in the appendix.
4. Please write all your answers in the Answer Booklet provided. **Show all relevant steps** to obtain maximum marks.

Question 1

a) Consider a population of four numbers 2, 3, 6, 9.

i) Calculate the **mean** and the **variance** of the four numbers. [2 marks]

ii) Two numbers, x and y, are to be added to this set of four numbers, such that the mean is increased by 1 and variance is increased by 2.5. It is known that $x > y$. Find x and y. [7 marks]

b) The following sample data were collected from a survey:

17.5	17.3	23.7	37.6	19.3	25.0	19.1	41.7	16.9	22.9
18.0	24.3	16.5	19.7	20.0	17.2	25.2	24.0	17.2	18.2
36.8	47.7	25.1	21.4	16.9	20.4	23.2	16.8	24.1	25.4
31.7	38.5	17.4	28.6	25.2	20.1	25.9	26.8	35.2	35.4
31.7	17.0	18.0	21.6	19.8	29.1	24.0	31.4	19.1	25.5

i) Copy and complete the frequency distribution table below:

Class Limits	Class Boundaries	Class midpoints, m	Tally	Frequency, f	mf	m^2f
16.5 – 20.9	16.45 – 20.95	18.7				
21.0 – 25.4	20.95 – 25.45	23.2				
25.5 – 29.9	25.45 – 29.95	27.7				
30.0 – 34.4	29.95 – 34.45	32.2				
34.5 – 38.9	34.45 – 38.95	36.7				
39.0 – 43.4	38.95 – 43.45	41.2				
43.5 – 47.9	43.45 – 47.95	45.7				

[4 marks]

ii) Calculate the estimated **mean** and **standard deviation** of the above grouped data. [3 marks]

iii) Draw a **histogram** from the frequency distribution table. Based on the histogram you drew, is the frequency distribution symmetrical, skewed to the left, or skewed to the right? [4 marks]

Question 2

a) In a small college, all 150 students are required to take up at least one martial arts class from a choice of Silat, Karate and Taekwondo. Some information about the number of students for the martial arts classes are shown in the table below:

Martial arts class(es)	Number of students
Silat	100
Silat and Karate	40
Karate and Taekwondo	40
Silat and Taekwondo	30
Silat, Karate and Taekwondo	10

It is also known that the number of students taking up Karate is 10 more than the number of students taking up Taekwondo.

i) Draw and complete a **Venn diagram** representing the above information. [4 marks]

ii) What is the **percentage** of students who take up at least two martial arts classes? [2 marks]

iii) What is the **probability** that a student from the college does not take Taekwondo classes? [2 marks]

iv) A student is randomly chosen. What is the **probability** that the student takes up Silat given that the student also takes up Karate? [2 marks]

b) There are 3 women and 4 men in the membership of a book club. A committee of 3 is to be chosen from the members of the club with the condition that the committee must consist of male and female members.

i) How many **ways** can a committee of 3 be formed with the above condition? [3 marks]

ii) It is later found out that one man and one woman in the book club are actually a married couple. The club's constitution mentions that the committee must not consist of members who are married to each other. However, it is acceptable if only one of them is in the committee and the other is not. How many **ways** can a committee of 3 be formed if this additional rule is enforced along with the first condition? [2 marks]

c) Consider the 7-letter word 'OPTICAL'.

i) How many **ways** can the letters be arranged if they must start with a consonant? [2 marks]

ii) How many different 4-letter arrangements can be made from the letters of the word if they must contain equal number of consonants and vowels? [3 marks]

Question 3

a) A sample of 820 adults showed that 80 of them had no credit cards, 116 had one card each, 94 had two cards each, 77 had three cards each, 43 had four cards each, and 410 had five or more cards each.

i) Prepare a **relative frequency distribution table** as shown below. Round off values to 5 decimal places if needed.

Number of credit cards owned, x	Relative frequency

[6 marks]

ii) Suppose one adult is randomly selected from these 820 adults. Find the **probability** that this adult has at most three credit cards. [2 marks]

b) Suppose a continuous random variable T has a probability density function given as

$$f(t)=\begin{cases} k(1-t^2) & -1\le t\le 1 \\ 0 & \text{elsewhere} \end{cases}$$

i) Find the value of the constant k. [5 marks]

ii) Find $P(T\le\frac{1}{2})$ [4 marks]

iii) Find $E(T)$, the expected value of T. [3 marks]

Question 4

a) Suppose that 20% of all patients undergoing MRI testing require sedation due to claustrophobia. If five patients are selected at random,

i) Find the **probability** that at least three of these patients require sedation. [4 marks]

ii) Calculate the **standard deviation** of the number of patients that require sedation. [2 marks]

b) A small life insurance company has determined that on the average it receives 1 death claim per day. Find the **probability** that the company receives at least four death claims on a randomly selected week. [6 marks]

c) It is estimated that the balances on all Visa credit cards issued by the Bank of Connecticut have a mean of \$845 and a standard deviation of \$270. Assume that the balances on all these Visa cards follow a normal distribution.

i) Find the **probability** that a random Visa credit card issued by this bank has a balance between \$1000 and \$1400. [4 marks]

ii) What **percentage** of the Visa cards issued by this bank has a balance of \$750 or more? [4 marks]

Question 5

a) The delivery times for all food orders at a fast-food restaurant during the lunch hour are normally distributed with a mean of 6.7 minutes and a standard deviation of 2.1 minutes. Find the **probability** that the mean delivery time for a random sample of 16 such orders at this restaurant is

i) between 7 and 8 minutes, [5 marks]

ii) within 1 minute of the population mean. [4 marks]

b) 43% of the U.S. population of voters voted for Bill Clinton in 1992. A sample of 100 voters was surveyed to see if they voted for Clinton in 1992.

i) What is the **probability** that more than half of the 100 people surveyed voted for Clinton in 1992? [6 marks]

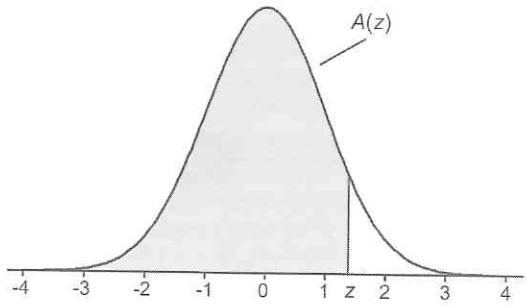
ii) Let $\hat{p}$ be the **proportion** of the people surveyed who voted for Clinton. The probability that $\hat{p}$ is less than some value h is 0.975. Find the value of h. [5 marks]

End of Paper

Appendix

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							